Solutions for HW NO 2

**3.18.** (a) Is there a difference in conductivity due to coating type? Use *α* = 0.05.

Yes, there is a difference in means. Refer to the Design-Expert output below..

**Design Expert Output**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 844.69 3 281.56 14.30 0.0003 significant

*A* *844.69* *3* *281.56* *14.30* *0.0003*

Residual 236.25 12 19.69

*Lack of Fit* *0.000* *0*

*Pure Error* *236.25* *12* *19.69*

Cor Total 1080.94 15

The Model F-value of 14.30 implies the model is significant. There is only

a 0.03% chance that a "Model F-Value" this large could occur due to noise.

**Treatment Means (Adjusted, If Necessary)**

**Estimated** **Standard**

**Mean** **Error**

1-1 145.00 2.22

2-2 145.25 2.22

3-3 132.25 2.22

4-4 129.25 2.22

**Mean** **Standard** **t for H0**

**Treatment** **Difference** **DF** **Error** **Coeff=0** **Prob > |t|**

1 vs 2 -0.25 1 3.14 -0.080 0.9378

1 vs 3 12.75 1 3.14 4.06 0.0016

1 vs 4 15.75 1 3.14 5.02 0.0003

2 vs 3 13.00 1 3.14 4.14 0.0014

2 vs 4 16.00 1 3.14 5.10 0.0003

3 vs 4 3.00 1 3.14 0.96 0.3578

1. Estimate the overall mean and the treatment effects.



1. Compute a 95 percent interval estimate of the mean of coating type 4. Compute a 99 percent interval estimate of the mean difference between coating types 1 and 4.

Treatment 4:  🡪 

Treatment 1 - Treatment 4: 🡪 

(d) Test all pairs of means using the Fisher LSD method with *α*=0.05.

Refer to the Design-Expert output above. The Fisher LSD procedure is automatically included in the output.

The means of Coating Type 2 and Coating Type 1 are not different. The means of Coating Type 3 and Coating Type 4 are not different. However, Coating Types 1 and 2 produce higher mean conductivity than does Coating Types 3 and 4.

1. Use the graphical method discussed in Section 3.5.3 to compare the means. Which coating produces the highest conductivity?

 Coating types 1 and 2 produce the highest conductivity.



1. Assuming that coating type 4 is currently in use, what are your recommendations to the manufacturer? We wish to minimize conductivity.

Since coatings 3 and 4 do not differ, and as they both produce the lowest mean values of conductivity, use either coating 3 or 4. As type 4 is currently being used, there is probably no need to change.

**3.19.** Reconsider the experiment in Problem 3.18. Analyze the residuals and draw conclusions about model adequacy.

There is nothing unusual in the normal probability plot. A funnel shape is seen in the plot of residuals versus predicted conductivity indicating a possible non-constant variance.





**3.27.** (a) Do chemists differ significantly? Use *α* = 0.05.

There is no significant difference at the 5% level, but chemists differ significantly at the 10% level.

Design Expert Output

**Response:** **Methyl Alcohol** **in %**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 1.04 3 0.35 3.25 0.0813 not significant

*A* *1.04* *3* *0.35* *3.25* *0.0813*

Residual 0.86 8 0.11

*Lack of Fit* *0.000* *0*

*Pure Error* *0.86* *8* *0.11*

Cor Total 1.90 11

The Model F-value of 3.25 implies there is a 8.13% chance that a "Model F-Value"

this large could occur due to noise.

**Treatment Means (Adjusted, If Necessary)**

**Estimated** **Standard**

**Mean** **Error**

1-1 84.47 0.19

2-2 85.05 0.19

3-3 84.79 0.19

4-4 84.28 0.19

**Mean** **Standard** **t for H0**

**Treatment** **Difference** **DF** **Error** **Coeff=0** **Prob > |t|**

1 vs 2 -0.58 1 0.27 -2.18 0.0607

1 vs 3 -0.32 1 0.27 -1.18 0.2703

1 vs 4 0.19 1 0.27 0.70 0.5049

2 vs 3 0.27 1 0.27 1.00 0.3479

2 vs 4 0.77 1 0.27 2.88 0.0205

3 vs 4 0.50 1 0.27 1.88 0.0966

1. Analyze the residuals from this experiment.: There is nothing unusual about the residual plots.





1. If chemist 2 is a new employee, construct a meaningful set of orthogonal contrasts that might have been useful at the start of the experiment.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Chemists | Total | C1 | C2 | C3 |
| 1 | 253.41 | 1 | -2 | 0 |
| 2 | 255.16 | -3 | 0 | 0 |
| 3 | 254.36 | 1 | 1 | -1 |
| 4 | 252.85 | 1 | 1 | 1 |
|  | Contrast Totals: | -4.86 | 0.39 | -1.51 |



🡪 Only contrast 1 is significant at 5%.

**3.31** (a) Is there significant variation in the calcium content from batch to batch? Use *α*=0.05.

Based on the ANOVA in the JMP output below, the batches differ significantly.

JMP Output

**Summary of Fit**

|  |  |
| --- | --- |
| RSquare | 0.525399 |
| RSquare Adj | 0.430479 |
| Root Mean Square Error | 0.066182 |
| Mean of Response | 23.4436 |
| Observations (or Sum Wgts) | 25 |

**Analysis of Variance**

| **Source** | **DF** | **Sum of Squares** | **Mean Square** | **F Ratio** |
| --- | --- | --- | --- | --- |
| Model | 4 | 0.09697600 | 0.024244 | 5.5352 |
| Error | 20 | 0.08760000 | 0.004380 | **Prob > F** |
| C. Total | 24 | 0.18457600 |  | 0.0036\* |

**Effect Tests**

| **Source** | **Nparm** | **DF** | **Sum of Squares** | **F Ratio** | **Prob > F** |  |
| --- | --- | --- | --- | --- | --- | --- |
| Batch | 4 | 4 | 0.09697600 | 5.5352 | 0.0036\* |  |

1. Estimate the components of variance.



This is verified in the JMP REML analysis shown below.

**JMP Output**

**Parameter Estimates**

| **Term** |  | **Estimate** | **Std Error** | **DFDen** | **t Ratio** | **Prob>|t|** |
| --- | --- | --- | --- | --- | --- | --- |
| Intercept |  | 23.4436 | 0.031141 | 4 | 752.82 | <.0001\* |

**REML Variance Component Estimates**

| **Random Effect** | **Var Ratio** | **Var Component** | **Std Error** | **95% Lower** | **95% Upper** | **Pct of Total** |
| --- | --- | --- | --- | --- | --- | --- |
| Batch | 0.907032 | 0.0039728 | 0.0034398 | -0.002769 | 0.0107147 | 47.562 |
| Residual |  | 0.00438 | 0.0013851 | 0.0025637 | 0.0091338 | 52.438 |
| Total |  | 0.0083528 |  |  |  | 100.000 |

**Covariance Matrix of Variance Component Estimates**

| **Random Effect** | **Batch** | **Residual** |
| --- | --- | --- |
| Batch | 1.1832e-5 | -3.837e-7 |
| Residual | -3.837e-7 | 1.9184e-6 |

1. Find a 95 percent confidence interval for 



1. Analyze the residuals from this experiment. Are the analysis of variance assumptions are satisfied?

🡪 The plot of residuals vs. predicted show no concerns.



The residuals used in the plot below are based on the REML analysis and shows no concerns. Note, normality is not a concern for this analysis.



**3.36.** Consider testing the equality of the means of two normal populations, where the variances are unknown but are assumed to be equal. The appropriate test procedure is the pooled *t* test. Show that the pooled *t* test is equivalent to the single factor analysis of variance.

 assuming *n*1 = *n*2 = *n*

** for a=2

Furthermore, , which is exactly the same as SSTreatments in a one-way classification with a=2. Thus we have shown that . In general, we know that  so that . Thus the square of the test statistic from the pooled *t*-test is the same test statistic that results from a single-factor analysis of variance with a=2.

**3.38.** In a fixed effects experiment, suppose that there are *n* observations for each of four treatments. Let  be single-degree-of-freedom components for the orthogonal contrasts. Prove that .





 and since

, we have 

for a=4.

**4.11.**

(a) Is there evidence to support the claim that the treatment means differ?

🡪 The ANOVA below identifies the treatment means are significantly different.

**Design Expert Output**

**Response** **Gene Expression**  
  **ANOVA for selected factorial model**  
 **Analysis of variance table [Classical sum of squares - Type II]**  
 **Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 9.206E+005 9 1.023E+005

Model 5.384E+005 2 2.692E+005 3.68 0.0457 significant  
  *A-Treatment* *5.384E+005* *2* *2.692E+005* *3.68* *0.0457*  
 Residual 1.316E+006 18 73130.15  
 Cor Total 2.775E+006 29  
  
 The Model F-value of 3.68 implies the model is significant. There is only  
 a 4.57% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 270.43 R-Squared 0.2903  
 Mean 293.82 Adj R-Squared 0.2114  
 C.V. % 92.04 Pred R-Squared -0.9714

PRESS 3.657E+006 Adeq Precision 5.288

**Treatment Means (Adjusted, If Necessary)**  
 **Estimated** **Standard**  
 **Mean** **Error**

1-MP Only 237.58 85.52

2-MP + HDMTX 478.62 85.52  
 3-MP + LDMTX 165.25 85.52

**Mean** **Standard** **t for H0**   
 **Treatment** **Difference** **df** **Error** **Coeff=0** **Prob > |t|**  
 1 vs 2 -241.04 1 120.94 -1.99 0.0616  
 1 vs 3 72.33 1 120.94 0.60 0.5572  
 2 vs 3 313.37 1 120.94 2.59 0.0184

(b) Check the normality assumption. Can we assume these samples are from normal populations?

🡪 The normal plot of residuals below identifies a slightly non-normal distribution.

****

(c) The ANOVA for the natural log transformed data identifies the treatment means as only moderately different with an *F* value of 0.07

Design Expert Output

**Response**  **Gene Expression**  
 **Transform:** **Natural Log** **Constant:** **0**  
  **ANOVA for selected factorial model**  
 **Analysis of variance table [Classical sum of squares - Type II]**  
 **Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 14.75 9 1.64

Model 6.30 2 3.15 3.09 0.0700   
  *A-Treatment* *6.30* *2* *3.15* *3.09* *0.0700*  
 Residual 18.32 18 1.02  
 Cor Total 39.37 29  
  
 The Model F-value of 3.09 implies there is a 7.00% chance that a "Model F-Value"   
 this large could occur due to noise.

Std. Dev. 1.01 R-Squared 0.2558  
 Mean 5.09 Adj R-Squared 0.1731  
 C.V. % 19.83 Pred R-Squared -1.0672

PRESS 50.89 Adeq Precision 4.942

**Treatment Means (Adjusted, If Necessary)**  
 **Estimated** **Standard**  
 **Mean** **Error**

1-MP Only 4.79 0.32

2-MP + HDMTX 5.73 0.32  
 3-MP + LDMTX 4.74 0.32

**Mean** **Standard** **t for H0**   
 **Treatment** **Difference** **df** **Error** **Coeff=0** **Prob > |t|**  
 1 vs 2 -0.95 1 0.45 -2.10 0.0505  
 1 vs 3 0.050 1 0.45 0.11 0.9122  
 2 vs 3 1.00 1 0.45 2.21 0.0405

(d) Analyze the residuals from the transformed data and comment on model adequacy.

🡪 The residual plots below identify no concerns with the model adequacy.





**4.28.** Consider the gene expression experiment in Problem 4.11. Assume that the subjects used in this experiment are random. Estimate the block variance component

The block variance component is:

